

# Pre Calc GTT

## Notes:

- The use of a calculator is permitted.
- Unless otherwise specified, if a decimal approximation is used, it must be accurate to three places after the decimal point.

## Reference Information:

Quadratic Formula: If  $ax^2 + bx + c = 0$ , then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

Factoring:  $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$        $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

## Laws of Exponents:

$$(a^m)^n = a^{mn} \quad \frac{a^n}{a^m} = a^{n-m} \quad (ab)^n = a^n b^n \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad a^{-n} = \frac{1}{a^n}$$
$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ or } (\sqrt[n]{a})^m$$

## Compound Interest:

Continuously:  $A = Pe^{rt}$        $n$  times per year:  $A = P\left(1 + \frac{r}{n}\right)^{nt}$

## Arithmetic Sequence and Series:

$$a_n = a_1 + (n-1)d \quad S_n = n\left(\frac{a_1 + a_n}{2}\right) = n\left(\frac{2a_1 + (n-1)d}{2}\right)$$

## Geometric Sequence and Series:

$$a_n = a_1 r^{n-1} \quad S_n = a_1 \left(\frac{1-r^n}{1-r}\right)$$

Conic Sections – GT ONLY

<b>Circle</b>	$x^2 + y^2 = r^2$	Center (0,0)	radius $r$
<b>Parabola</b>	$x^2 = 4py$ or $y = \frac{1}{4p}x^2$ opens up if $p > 0$ , opens down if $p < 0$	Vertex (0,0)	
	$y^2 = 4px$ or $x = \frac{1}{4p}y^2$ opens right if $p > 0$ , opens left if $p < 0$	Vertex (0,0)	
	$p$ is the distance from the vertex to the focus and from the vertex to the directrix		
<b>Ellipse</b>	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	major axis horizontal	Center (0,0) $a > b$
	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	major axis vertical	Center (0,0) $a > b$
	$a = \text{center to vertex}$ , $c = \text{center to focus}$ $c^2 = a^2 - b^2$		
<b>Hyperbola</b>	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	transverse axis horizontal	Center (0,0)
	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	transverse axis vertical	Center (0,0)
	$a = \text{center to vertex}$ $c = \text{center to focus}$ $c^2 = a^2 + b^2$		

<b>Circle</b>	$(x-h)^2 + (y-k)^2 = r^2$	Center $(h, k)$	radius $r$
<b>Parabola</b>	$(x-h)^2 = 4p(y-k)$ or $y-k = \frac{1}{4p}(x-h)^2$ opens up if $p > 0$ , opens down if $p < 0$ Vertex $(h, k)$  $(y-k)^2 = 4p(x-h)$ or $x-h = \frac{1}{4p}(y-k)^2$ opens right if $p > 0$ , opens left if $p < 0$ Vertex $(h, k)$  <p><math>p</math> is the distance from the vertex to the focus or vertex to the directrix</p>		
<b>Ellipse</b>	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ major axis horizontal    Center $(h, k)$ $a > b$  $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ major axis vertical    Center $(h, k)$ $a > b$  <p><math>a</math> = center to vertex, <math>c</math> = center to focus    <math>c^2 = a^2 - b^2</math></p>		
<b>Hyperbola</b>	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ transverse axis horizontal    Center $(h, k)$  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ transverse axis vertical    Center $(h, k)$  <p><math>a</math> = center to vertex    <math>c</math> = center to focus    <math>c^2 = a^2 + b^2</math></p>		

Name \_\_\_\_\_

**USE A SEPARATE SHEET OF PAPER AND SHOW ALL WORK.  
PROBLEMS WITH AN ASTERISK \* ARE FOR GT ONLY.**

**I. Polynomials and operations on real and imaginary numbers.**

**A. Simplify these expressions**

1.  $\sqrt{-100}$

2.  $\sqrt{-4 \cdot -9}$

3.  $(i\sqrt{7})^2$

4.  $\sqrt[3]{2x} \cdot \sqrt[3]{4x^2y^2} \cdot \sqrt[3]{2y^4}$

5.  $(3 + 2i) + (5 + 7i)$

6.  $2i(3 - i)$

7.  $(3 + 2i)(3 - 2i)$

8.  $(3 + i\sqrt{5})^2$

9.  $\frac{8}{-2i}$

10.  $-\sqrt{-9}$

\*11.  $\frac{5i}{6-2i}$  (Hint: Use the conjugate of denominator)

**B. Factor Completely**

1.  $t^2 - 4t - 21$

2.  $8x^3 - 1$  hint: use formula

3.  $x^3 + 125$  hint: use formula

\*4.  $x^3 - 2x^2 - 4x + 8$  Hint: grouping

**C. Simplify the following expressions.**

1.  $5x^2 \cdot 2x^5$

2.  $(-2c^3)^2$

3.  $t^3 \cdot t^{(n-3)}$

4.  $\frac{10 \cdot 2^6}{8 \cdot 2^{-2}}$

\*5.  $(x^m)^n \cdot (x^n)^{n-m}$

**D. Divide and simplify these expressions. Use either polynomial long division or synthetic division.**

1.  $\frac{x^2 + 2x - 3}{x + 3}$

**E. Solve each quadratic equation for x using either factoring techniques, or the quadratic formula.**

1.  $(x - 1)(x + 3) = 0$

2.  $x(x - 4) = 2(4 - x)$

3.  $x^2 + 4x = -3$

4.  $2x^2 - 32x = 0$

**F. Graph the functions using a calculator. Sketch it on your paper. Describe the following characteristics for each function:**

- a. domain and range    b. zeros    c. y-intercept    d. end behavior  
e. intervals where the function is increasing and/or decreasing

1.  $f(x) = x^3 - 3x^2 + x + 1$

2.  $f(x) = x^2 + 2x + 1$

## II. Function Operations

**A. If  $f(x) = x^2 - 4$  and  $g(x) = \sqrt{2x + 4}$ , determine**

1.  $f(3)$

2.  $f(x) = 0$  when  $x = ?$

3.  $f(g(4))$

4.  $f(g(x))$

5. Domain of  $f(g(x))$

6.  $g(f(0))$

\* 7.  $g(f(a + 2))$

8.  $f^{-1}(x)$

## III. Rational Expressions and Rational Functions

**A. Graph the following functions using a table of 5 values. Also identify:**

a. domain

b. range

c. asymptotes

1.  $f(x) = \frac{2x}{x + 4}$

2.  $h(x) = \frac{3}{x + 1} - 2$

3.  $k(x) = \frac{4}{x - 2}$

**B. Simplify. Write your answer as a single fraction.**

1.  $\frac{3x^2 + 6x^3}{9x}$

2.  $\frac{x^2 - 25}{x^2 + 7x + 10}$

3.  $\frac{2x}{x+5} \div \frac{6x^2}{2x+10}$

4.  $\frac{\frac{3}{x+2}}{\frac{6}{x}}$

5.  $\frac{x-2}{x} + \frac{x+4}{2x}$

6.  $\frac{4x}{x+6} + 2$

#### IV. Rewriting and Solving Equations

##### A. Solve each equation for y.

1.  $7y + 6x = 10$

2.  $\frac{1}{4}y - 7x = \frac{15}{2}$

##### B. Find the solution(s) of the given systems of equations. Write answers in the form (x, y).

1.  $-2x - 5y = 7$

2.  $4x + 9y = 2$

$7x + y = -8$

$2x + 6y = 1$

##### C. Solve for x and y.

1.  $\begin{cases} x + 9y = 9 \\ 3x + 6y = 6 \end{cases}$

#### V. Pythagorean Theorem.

##### A. Solve for the missing side of the triangle using the Pythagorean Theorem $a^2 + b^2 = c^2$ :

1.  $a = 6$  ft.  $b = 8$  ft.

2.  $b = 17$  ft.  $c = 19$  ft.

